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$|V_{ub}|$ and Constraints on the Leading-Twist Pion Distribution Amplitude from $B \to \pi \ell \nu$

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Abstract:

Using new experimental data on the leptonic mass spectrum of $B \to \pi \ell \nu$, we simultaneously determine $|V_{ub}|$ and constrain a_2^π and a_4^π, the first two Gegenbauer moments of the pion's leading-twist distribution amplitude. We find $|V_{ub}| = (3.2 \pm 0.1 \pm 0.1 \pm 0.3) \times 10^{-3}$, where the first error is experimental, the second comes from the shape of the form factor in q^2 and the third is a 8% uncertainty from the normalisation of the form factor. We also find $a_2^\pi(1\,\text{GeV}) = 0.19 \pm 0.19$ and $a_4^\pi(1\,\text{GeV}) \geq -0.07$.

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$|V_{ub}|$ is one of the least well-known elements of the CKM quark-mixing matrix. A more precise determination of this parameter will not only greatly improve the constraints on the unitarity triangle, but also provide a stringent test of the CKM mechanism of flavour structure and CP violation. In this letter, we determine $|V_{ub}|$ from the exclusive semileptonic decay $B \to \pi\ell\nu$, based on the invariant lepton-mass spectrum recently reported by BaBar [1] and the light-cone sum rule calculations of the relevant form factor in Ref. [2].

The hadronic matrix element relevant for $B \to \pi\ell\nu$ is given by

$$\langle\pi(p_\pi)|\bar{u}\gamma_\mu b|B(p_B)\rangle = \left(p_B + p_\pi - q\,\frac{m_B^2 - m_\pi^2}{q^2}\right)_\mu f_+(q^2) + \frac{m_B^2 - m_\pi^2}{q^2}\,q_\mu\,f_0(q^2), \qquad (1)$$

where the form factors $f_{+,0}$ depend on $q^2 \equiv (p_B - p_\pi)^2$, the invariant mass of the lepton-pair, with $0\,\mathrm{GeV}^2 \leq q^2 \leq (m_B - m_\pi)^2 = 26.4\,\mathrm{GeV}^2$. f_+ is the dominant form factor, i.e. the only one needed for calculating the spectrum in q^2,

$$\frac{d\Gamma}{dq^2}\,(B^0 \to \pi^-\ell^+\nu_\ell) = \frac{G_F^2|V_{ub}|^2}{192\pi^3 m_B^3}\,\lambda^{3/2}(q^2)|f_+(q^2)|^2 \qquad (2)$$

for massless leptons; $\lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2$ is the usual phase-space factor. The determination of $|V_{ub}|$ from $B \to \pi\ell\nu$ requires theoretical input on f_+, which has been the subject of many a calculation using various methods, in particular quark models [3], QCD sum rules on the light-cone (LCSRs) [2, 4, 5] and lattice simulations [6]. The challenge for theory is twofold: the region of applicability of theoretical calculations is, in most cases, restricted to part of the full physical phase-space; a calculation of f_+ for, say, small values of q^2 is, however, not sufficient, as experimental data on the decay spectrum are still very scarce, so that any meaningful extraction of $|V_{ub}|$ necessitates the extrapolation of the form factor to all q^2. LCSR calculations, for instance, are valid for large pion momentum, which translates into small to moderate $q^2 \lesssim 14\,\mathrm{GeV}^2$, whereas lattice calculations are restricted to small pion momentum, corresponding to large $q^2 \gtrsim 15\,\mathrm{GeV}^2$. Extrapolations rely either on a model for the q^2-dependence of f_+, like vector meson dominance or the parametrisation advocated by Becirevic and Kaidalov [7], or dispersive bounds on the form factor, which have been studied for instance in Ref. [8]. The BaBar collaboration has measured the spectral decay distribution in 5 bins in q^2 [1], which is a significant improvement over previous results reported for 3 bins [9], and allows one, for the first time, to assess the validity of various parametrisations of the q^2-dependence of f_+ like

- vector meson dominance (VMD);
- the parametrisation of Becirevic and Kaidalov (BK) [7];
- the extended BK parametrisation used by Ball and Zwicky (BZ) [2].

All these parametrisations can be motivated from the exact representation of f_+ in terms of a dispersion relation,

$$f_+(q^2) = \frac{\mathrm{Res}_{q^2=m_{B^*}^2} f_+(q^2)}{q^2 - m_{B^*}^2} + \frac{1}{\pi}\int_{(m_B+m_\pi)^2}^{\infty} dt\,\frac{\mathrm{Im}\,f_+(t)}{t - q^2 - i\epsilon}\,, \qquad (3)$$

where $m_{B^*} = 5.325\,\mathrm{GeV}$ is the mass of the $B^*(1^-)$ meson which induces a pole below the lowest multiparticle threshold at $q^2 = (m_B + m_\pi)^2$. Vector meson dominance assumes that the form factor is dominated, for all physical q^2, by the first term in (3):

$$f_+(q^2)\big|_{\mathrm{VMD}} = \frac{f_+(0)}{1 - q^2/m_{B^*}^2}\,; \tag{4}$$

$f_+(0)$ is the only free parameter of the VMD parametrisation. Becirevic and Kaidalov suggested as an alternative parametrisation the replacement of the second term in (3) by an effective pole at higher mass,

$$f_+(q^2)\big|_{\mathrm{BZ}} = \frac{r_1}{1 - q^2/m_{B^*}^2} + \frac{r_2}{1 - \alpha\, q^2/m_{B^*}^2} \equiv \frac{f_+(0)}{1 - q^2/m_{B^*}^2} + \frac{r\, q^2/m_{B^*}^2}{(1 - q^2/m_{B^*}^2)(1 - \alpha\, q^2/m_{B^*}^2)} \tag{5}$$

with the three parameters (r_1, r_2, α) or $(f_+(0), r, \alpha)$, which are related by

$$f_+(0) = r_1 + r_2, \qquad r = r_2(\alpha - 1), \tag{6}$$

where $0 < \alpha < 1$ parameterises the position of the effective pole. This parametrisation was used by BZ to describe the results from LCSRs [2], whereas Becirevic and Kaidalov, faced with the challenge to fit three independent parameters to lattice data with limited accuracy, implemented the additional constraint $r \equiv \alpha f_+(0)$ motivated from heavy quark expansion, and obtained the following expression in terms of two parameters, (c_B, α) or $(f_+(0), \alpha)$:

$$f_+(q^2)\big|_{\mathrm{BK}} = \frac{c_B(1-\alpha)}{(1 - q^2/m_{B^*}^2)(1 - \alpha\, q^2/m_{B^*}^2)} \equiv \frac{f_+(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha\, q^2/m_{B^*}^2)}\,. \tag{7}$$

The BaBar collaboration has measured the integrated q^2-spectrum of $B \to \pi \ell \nu$ in five bins in q^2 [1], which allows one to confront the above parametrisations with experiment. Fitting the data by the VMD formula (4), we find $\chi^2 = 9.2/4$ d.o.f. The BK parametrisation (7) fits the data with $\chi^2 = 3.5/3$ d.o.f. and $\alpha = 0.61 \pm 0.09$. Fitting the BZ parametrisation (5) is slightly more subtle, as the data prefer $\alpha \to 1$ or even larger, which is outside the allowed parameter-space.[1] Bounding $\alpha \leq 1$, we find a minimum $\chi^2 = 1.65/2\,$d.o.f. and $r/f_+(0) = 0.24 \pm 0.08$, $\alpha = 1.00^{+0}_{-0.15}$. While this implies that the VMD parametrisation is disfavoured,[2] both BK and BZ are viable parametrisations. Motivated by these results, we formulate the following strategy for extracting $|V_{ub}|$ from the data: we

- calculate f_+ from LCSRs for values of q^2 where the method is applicable;
- extrapolate the results to all q^2 using the experimentally favoured BK and BZ parametrisations;
- use the experimental information on the spectrum to constrain the input parameters of the LCSRs, in particular the leading-twist π distribution amplitude;

[1]For $\alpha = 1$ the rescaling (6) is no longer valid.

[2]Actually VMD is disfavoured not only from the experimental point of view, but also from the theoretical one, as the values of $f_+(0)$ do not agree with indepedent determinations of the residue of the B^* pole, see the discussion in Sec. 4 of Ref. [2].

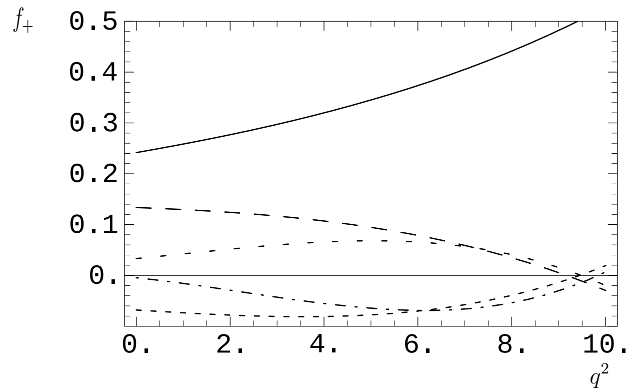

Figure 1: Respective size of different contributions in a_n^π to $f_+(q^2)$ as a function of q^2 for central values of the LCSR input parameters. Solid line: $a_n^\pi = 0$. Remaining lines from top to bottom (at $q^2 = 0$): contributions proportional to a_2^π, a_6^π, a_8^π, a_4^π.

- determine $|V_{ub}|$ from the total branching ratio using the BK and BZ parametrisations of f_+.

Let us start with the calculation of f_+. In Ref. [2], we have presented a comprehensive analysis of $B \to (\pi, K, \eta)$ decay form factors calculated from QCD sum rules on the light-cone, to $O(\alpha_s)$ accuracy for twist-2 and the dominant twist-3 contributions; earlier analyses can be found in Refs. [4, 5]. We refer to these papers for an explanation of the method. The main theoretical uncertainty of these analyses comes from the pion's leading-twist light-cone distribution amplitude (DA) ϕ_π; other sources of uncertainty include the b quark mass, the quark condensate and sum rule specific parameters (Borel parameter and continuum threshold). The resulting total uncertainty of f_+ is between 10% and 13% [2]. Whereas the other parameters mainly determine the normalisation of the form factor, ϕ_π affects also and in particular the q^2-dependence and hence can be constrained from the measured spectrum. The DA is usually expressed in terms of its conformal expansion,

$$\phi_\pi(u, \mu) = 6u(1-u)\left(1 + \sum_{n=1}^{\infty} a_{2n}^\pi(\mu) C_{2n}^{3/2}(2u-1)\right), \tag{8}$$

where u is the momentum fraction of the quark in the π and runs from 0 to 1. The $C_n^{3/2}$ are Gegenbauer polynomials and a_n, the so-called Gegenbauer moments, are hadronic parameters which depend on the factorisation scale μ. The respective contributions of $a_{n\leq 8}^\pi$ to f_+ are shown in Fig. 1, for a typical choice of input parameters. The plot reveals that the q^2-dependence of the form factor is mostly sensititive to a_2^π and only to a lesser extent to higher Gegenbauer-moments, which agrees with the findings of Ref. [10]. We hence decide against using the models for ϕ_π proposed in Ref. [10], but stick with the expansion (8), which we truncate after the contribution in a_4^π. The values of the lowest lying Gegenbauer moments $a_{2,4}^\pi$ have been constrained from various sources, cf. Ref. [11, 12, 13], but still come with rather large uncertainties. A very conservative range of allowed values consistent with all

known constraints is

$$0 \leq a_2^\pi(1\,\text{GeV}) \leq 0.3, \qquad -0.15 \leq a_4^\pi(1\,\text{GeV}) \leq 0.15. \tag{9}$$

In this letter, we aim to constrain $a_{2,4}^\pi$ from experimental data within the above range.

We obtain values for $f_+(q^2)$ in dependence on a_2^π, a_4^π and m_b using the following criteria for the evaluation of the LCSRs:

- we calculate $f_+(q^2)$ as a function of the Borel parameter M^2 and the continuum threshold s_0 for two different values of m_b, $m_b \in \{4.7, 4.8\}\,\text{GeV}$, five different values of q^2, $q^2 \in \{0, 2.5, 5, 7.5, 10\}\,\text{GeV}^2$, and 16 different values of $(a_2^\pi(1\,\text{GeV}), a_4^\pi(1\,\text{GeV}))$: $a_2^\pi \in \{0, 0.1, 0.2, 0.3\}$, $a_4^\pi \in \{-0.15, -0.05, 0.05, 0.15\}$;[3] we interpolate the results in a_2^π and a_4^π in order to obtain f_+ as a smooth function of these parameters;

- for each value of the input parameters, we determine f_+ at the minimum in the Borel parameter M^2, which implies that M^2 becomes mildly dependent on q^2. As discussed in Ref. [2], this procedure ensures that a LCSR for m_B, obtained from the derivative of the LCSR for f_+ in M^2, yields the physical value $m_B = 5.28\,\text{GeV}$;

- we choose the continuum threshold s_0 in such a way that the continuum contribution to the LCSR is constant for all q^2; this implies that also s_0 becomes (mildly) dependent on q^2. For each value of the input parameters, we calculate f_+ for three different values of the continuum contribution, 15%, 20% and 25%;

- the LCSR actually yields $f_B f_+$, f_B being the leptonic decay constant of the B meson; in order to extract f_+, we divide the LCSR by f_B as calculated from a QCD sum rule to the same accuracy in α_s.

Some of these criteria differ from those applied in Ref. [2]. We chose to modify the criteria used in our previous work since we focus, in this letter, on the dependence of the form factor on $a_{2,4}^\pi$, which can be meaningfully determined only if f_+ is calculated using exactly the same criteria for all values of q^2, m_b, a_2^π and a_4^π. It is for this reason that we require the continuum contribution to be the same for all input parameters. The drawback of this procedure is that the sum rule specific parameters M^2 and s_0 both become dependent on q^2, so that, in order to keep the calculational effort at a manageable level, we have to restrict ourselves to a few points in q^2. For each value of f_+ we calculate the theoretical uncertainty by varying

- M^2 by 40% around the central value;

- s_0 by $\pm 1\,\text{GeV}^2$;

- the central value 20% of the continuum contribution between 15% and 25%;

- the central value of the quark condensate, $-\langle \bar{q}q\rangle(1\,\text{GeV}) = (0.24\,\text{GeV})^3$, between $((0.24 \pm 0.01)\,\text{GeV})^3$.

[3] In the actual calculation, $a_{2,4}^\pi$ are scaled up to the factorisation scale $\mu = \sqrt{m_B^2 - m_b^2} \approx 2.2\,\text{GeV}$ using NLO evaluation.

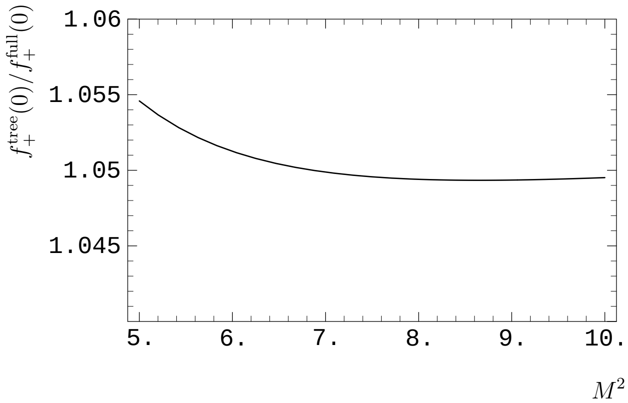

Figure 2: $f_+^{\rm tree}(0)/f_+^{\rm full}(0)$ as a function of the Borel parameter M^2 for $a_2^\pi = 0.1$, $a_4^\pi = 0$, $m_b = 4.8\,\text{GeV}$, $s_0^{\rm tree} = 35.25\,\text{GeV}^2$, $s_0^{\rm full} = 34.42\,\text{GeV}^2$, which corresponds to 20% continuum contribution. The ratio of the decay constants is $f_B^{\rm full}/f_B^{\rm tree} = 1.26$.

m_b [GeV]	4.7	4.8
f_B [MeV]	191 ± 6	164 ± 4

Table 1: f_B from a QCD sum rule to $O(\alpha_s)$ accuracy; the error is obtained by varying $(M^2)_{f_B}$ by 40% and $(s_0)_{f_B}$ by $\pm 1\,\text{GeV}^2$. The current world average from lattice calculations is $f_B = (189 \pm 27)\,\text{MeV}$ [14].

The above ranges of sum rule parameters are rather conservative and account for the "systematic" uncertainty of QCD sum rule calculations. All errors are added linearly, which yields a typical theoretical uncertainty of 8%. At this point we would like to comment on the treatment of f_B. The motivation for calculating f_B from a QCD sum rule instead of using, for instance, the current world average from lattice calculations [14], is that (a) f_B receives large $O(\alpha_s)$ and $O(\alpha_s^2)$ corrections from gluon-exchange diagrams that also enter the LCSR for $f_B f_+$ and (b) f_B is very sensitive to m_b. By dividing the LCSR for $f_B f_+$ by f_B obtained from a QCD sum rule to the same accuracy in α_s, and using the same value of m_b, one expects those large contributions to cancel and to reduce the sensitivity to the value of m_b. One can check the extent to which the cancellation takes place by comparing the results of our $O(\alpha_s)$ calculation with that at tree-level. We calculate $f_+^{\rm tree}(0) \equiv (f_B f_+(0))^{\rm tree}/(f_B)^{\rm tree}$ for $m_b = 4.8\,\text{GeV}$, $a_2^\pi = 0.1$, $a_4^\pi = 0$ using the same criteria as for the full form factor $f_+^{\rm full}(0)$ including $O(\alpha_s)$ corrections. In Fig. 2 we plot the ratio $f_+^{\rm tree}(0)/f_+^{\rm full}(0)$ for the respective optimum values s_0 as a function of M^2. The minima in M^2 are around $7\,\text{GeV}^2$. The ratio is nearly constant ~ 1.05, whereas the ratio of the decay constants, $f_B^{\rm full}/f_B^{\rm tree}$ is 1.26. This means that there is indeed a strong cancellation between the radiative corrections to the LCSR for $f_B f_+(0)$ and the QCD sum rule for f_B. We have checked that similar cancellations also occur for other values of m_b and nonzero q^2. Nonetheless there is a residual uncertainty due to the treatment of f_B which we estimate to be about half the difference between the value of f_B calculated from a QCD sum rule to $O(\alpha_s)$ accuracy and the central lattice value,

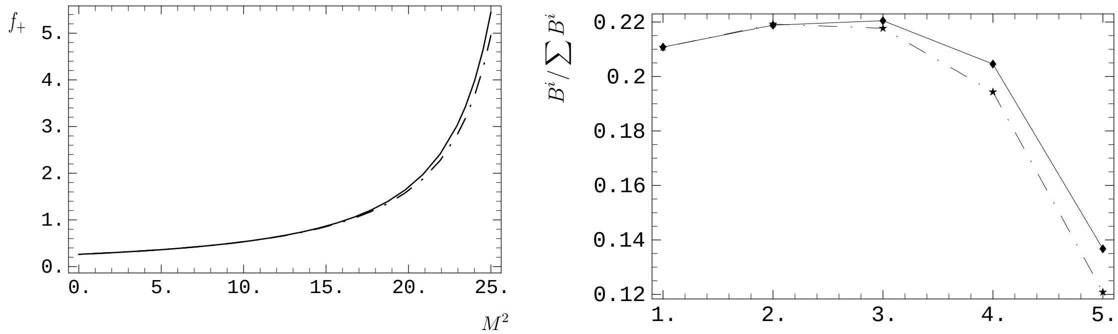

Figure 3: Left: $f_+(q^2)$ as a function of q^2 for $m_b = 4.8\,\text{GeV}$, $a_2^\pi = 0.115$ and $a_4^\pi = -0.015$ fitted to the BK parametrisation (7) (solid line) and to the BZ parametrisation (5) (dot-dashed line). Right: corresponding integrated spectra in the five q^2-bins used by BaBar.

i.e. about 5%. The QCD sum rule results for f_B are given, with errors, in Tab. 1. Adding these two errors linealy, we obtain an uncertainty of f_B of 8% which is independent of q^2 and translates into a 8% uncertainty of the normalisation of f_+, which we treat separately from the error of $f_B f_+$ calculated as described above.[4]

The next task is to compare the form factor predictions to data and to determine best-fit values of $a_{2,4}^\pi$, using the experimentally favoured BK and BZ parametrisations in order to extrapolate the LCSR results to all physical q^2. It turns out that the LCSR results are actually described extremely well by the BK and BZ parametrisations, to within better than 0.5%, as already noted in Ref. [2]. In Fig. 3 we show the difference between the BK and the BZ fit, both for the form factor and the integrated q^2-spectra in the ith bin, $5(i-1)\text{GeV}^2 \leq q^2 \leq 5i\text{GeV}^2$, $1 \leq i \leq 5$. The form factors start to noticeably deviate only for very large q^2. For the fit of the experimental spectrum, we treat the experimental errors as uncorrelated, but allow for a correlation of theory errors and perform the least-χ^2 fits using the following χ^2 function and error matrix E:

$$\chi^2 = \sum_{i,j=1}^{5} (B_i^{\text{th}} - B_i^{\text{exp}})(E^{-1})_{ij}(B_j^{\text{th}} - B_j^{\text{exp}}),$$

$$E_{ij} = (\sigma_i^{\text{exp}})^2\delta_{ij} + (\Delta B_i^{\text{th}})^2\delta_{ij} + \mathcal{C}^2(\Delta B_i^{\text{th}})(\Delta B_j^{\text{th}})(1-\delta_{ij}),$$

where B_i is the partial branching fractions ith q^2-bin, ΔB_i^{th} the corresponding theory error (without the error of the overall normalisation of f_+) and $\mathcal{C}$ is the correlation of theory errors which we vary within $0 \leq \mathcal{C} \leq 1$.

We first study the BK parametrisation which features one parameter that can be determined from the experimental spectrum: $\alpha = 0.61 \pm 0.09$. As f_+ depends on actually three parameters, a_2^π, a_4^π and m_b, only one of them can be constrained. In Tab. 2 we give the best-fit values of a_2^π for a_4^π and m_b fixed. The table reveals that it is indeed possible to reproduce the experimental central value of α for any given a_4^π and m_b. It is also obvious

[4]As the quark condensate is a common input parameter for both $f_B f_+$ and f_B, the effect of its variation is included only once, in the uncertainty of $f_B f_+$.

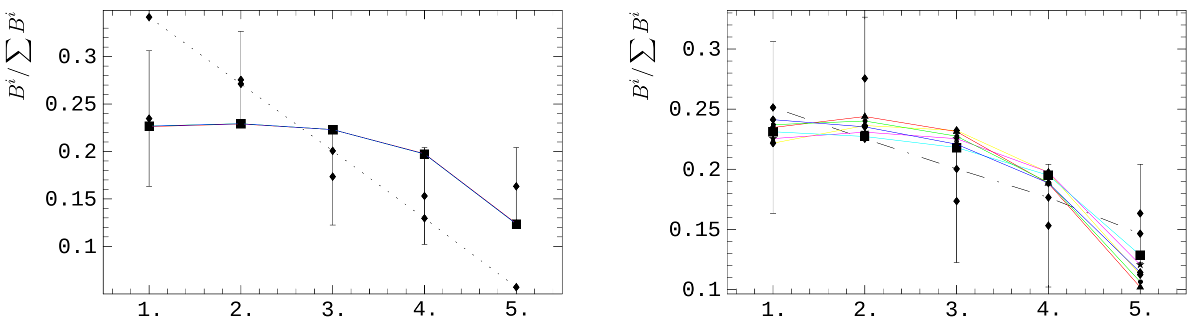

Figure 4: (Colour online) Best fit results for the partially integrated differential decay rates (solid lines) and experimental data (partially integrated spectra in q^2-bins 1 to 5). Left: BK parametrisation of the form factor, right: BZ parametrisation. Solid lines: set of input parameters as in Tabs. 2 and 3. Dotted line: VMD parametrisation, Eq. (4). Dash-dotted line: best BZ fit to experimental data.

that the impact of the precise value of a_4^π is much smaller than that of m_b. Averaging over a_4^π within $[-0.15, 0.15]$ and over m_b, we find

$$a_2^\pi(1\,\mathrm{GeV})|_{\mathrm{BK}} = 0.14^{+0.18}_{-0.20}\,. \tag{10}$$

We would like to stress that this is a completely new determination of a_2^π and agrees very well with other determinations of this parameter [11, 12, 13]. What is truly remarkable, however, is that, despite different best-fit values of a_2^π, the resulting values of $f_+(0)$, and hence $|V_{ub}|$, agree within 3%. That is: the theory error due to m_b and $a_{2,4}^\pi$ gets largely diminished by the constraints on the spectrum. Using the parameter sets in Tab. 2, we plot, in Fig. 4, the partially integrated spectra, normalised to the full branching ratio, together with the experimental data. All six parameter sets produce nearly the same curve which coincides with the best experimental fit using the BK parametrisation. Using the average value of the branching ratio as given by HFAG [15], $B(B^0 \to \pi^- \ell^+ \nu_\ell) = (1.36 \pm 0.11) \times 10^{-4}$, we obtain

$$|V_{ub}|_{\mathrm{BK}} = (3.2 \pm 0.1 \pm 0.1 \pm 0.3) \times 10^{-3}, \tag{11}$$

where the first error is experimental, the second comes from the uncertainty in the shape, due to the spread of values of $a_{2,4}^\pi$, and the third is from the normalisation of the form factor.

Let us now turn to the BZ parametrisation, which featurs two parameters that can be determined from the shape of the spectrum, $r/f_+(0) = 0.24 \pm 0.08$ and $\alpha = 1.00^{+0}_{-0.15}$, which allows one to constrain for instance a_2^π and a_4^π in dependence on m_b. The resulting constraints are shown in Fig. 5. The minimum χ^2 for (a_2^π, a_4^π) within the range specified in (9) is reached for $a_2^\pi = 0.23$, $a_4^\pi = 0.15$, i.e. at the border of the parameter space; fixing $a_4^\pi = 0.15$, the best-fit value of a_2^π is

$$a_2^\pi(1\,\mathrm{GeV})|_{\mathrm{BZ}} = 0.23 \pm 0.15 \quad \text{for } a_4^\pi(1\,\mathrm{GeV}) = 0.15. \tag{12}$$

The contours shown in the figure include all (a_2^π, a_4^π) for which the fit of the corresponding form factor to the data yields $\chi^2 \leq \chi^2_{\min} + 1 = 4.06$. We immediately read off the following

m_b	a_4^π	a_2^π	$f_+(0)$	α	$\lvert V_{ub}\rvert$
4.8 GeV	-0.15	0.19 ± 0.13	0.277	0.61	3.17×10^{-3}
	0	0.18 ± 0.14	0.272	0.61	3.23×10^{-3}
	0.15	0.16 ± 0.15	0.268	0.61	3.29×10^{-3}
4.7 GeV	-0.15	0.08 ± 0.13	0.274	0.61	3.21×10^{-3}
	0	0.11 ± 0.14	0.272	0.61	3.23×10^{-3}
	0.15	0.14 ± 0.16	0.271	0.61	3.25×10^{-3}

Table 2: $a_2^\pi(1\,\mathrm{GeV})$ from the fit to the shape of the experimental q^2-spectrum, for fixed m_b and a_4^π using the BK parametrisation (7). The table also gives the corresponding BK parameters $f_+(0)$ and α and the resulting central value of $|V_{ub}|$. $\chi^2_{\rm min} = 3.5/3\,$d.o.f.

m_b	a_4^π	a_2^π	$f_+(0)$	r	α	χ^2	$\lvert V_{ub}\rvert$
4.8 GeV	0	0.15	0.268	0.18	0.54	3.67	3.26×10^{-3}
	0.15	0.23	0.278	0.14	0.73	3.06	3.22×10^{-3}
4.7 GeV	0	0.10	0.270	0.18	0.52	3.71	3.26×10^{-3}
	0.15	0.24	0.282	0.14	0.70	3.11	3.23×10^{-3}

Table 3: Ditto for the BZ parametrisation (5). In addition we give the minimum χ^2, for 3 d.o.f., for the fit of the form factor to the data.

constraints:

$$a_2^\pi(1\,\mathrm{GeV}) \geq 0, \qquad a_4^\pi(1\,\mathrm{GeV}) \geq -0.07. \tag{13}$$

In Tab. 3, we give the fit results for $a_4^\pi = 0$ and 0.15. The best-fit parameters do not agree, for $a_{2,4}^\pi$ within the range specified in (9), with those favoured by experiment. We have already pointed out earlier that the experimentally favoured value $\alpha = 1$ is actually outside the theoretically allowed parameter space and is not supported by the results of the LCSR calculation. As before, we find that the different values for $a_{2,4}^\pi$ result in nearly the same values of $|V_{ub}|$. In Fig. 4 we plot the best-fit values for the partially integrated branching fractions, obtained from the parameter sets in Tab. 3, in comparison with the experimental results. For $|V_{ub}|$, we find

$$|V_{ub}|_{\rm BZ} = (3.2 \pm 0.1 \pm 0.1 \pm 0.3) \times 10^{-3} \tag{14}$$

which agrees with (11).

Combining the results from the BK and the BZ analyses, we get the following final result for $|V_{ub}|$:

$$|V_{ub}| = (3.2 \pm 0.1 \pm 0.1 \pm 0.3) \times 10^{-3}; \tag{15}$$

the first error in $|V_{ub}|$ is experimental, the second comes from the shape of the form factor and the third from the overall normalisation of f_+. As for the constraints on $a_{2,4}^\pi$, our final

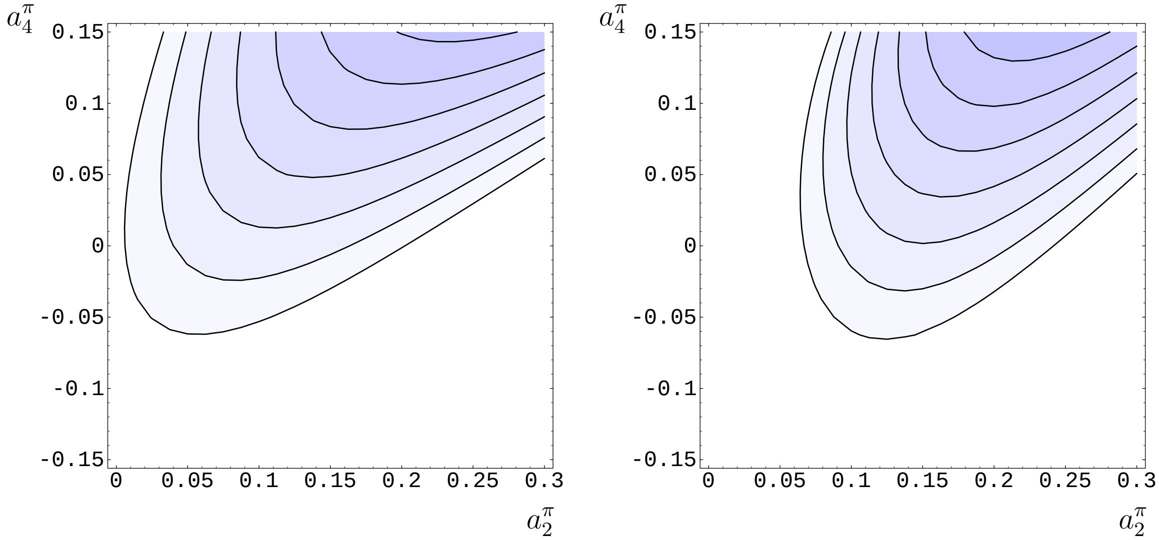

Figure 5: (Colour online) Allowed values of $a^\pi_{2,4}$ for $m_b = 4.7\,\mathrm{GeV}$ (left) and $m_b = 4.8\,\mathrm{GeV}$ (right). The minimum χ^2 is 3.06 and the contours include (a_2^π, a_4^π) with $\chi^2 \leq 4.06$.

results are

$$a_2^\pi(1\,\mathrm{GeV}) = 0.19 \pm 0.19, \quad a_4^\pi(1\,\mathrm{GeV}) \geq -0.07. \tag{16}$$

The value of a_2^π is the weighted average of (10) and (12). We would like to stress again that these values result from a new and independent determination of these parameters, which is both consistent with and complementary to the results found in Refs. [11, 12, 13].

To summarize, we have discussed the constraints posed by recent experimental data on the shape of the $B \to \pi$ decay form factor f_+. We have found that both the BK parametrisation Eq. (7) and the BZ parametrisation Eq. (5) can describe the data, whereas vector meson dominance is disfavoured. We have calculated f_+ from QCD sum rules on the light-cone for small to moderate q^2 and extrapolated the form factor to all physical q^2 using the BK and BZ parametrisations, respectively. We have then used the experimental data to constrain the pion distribution amplitude ϕ_π that enters the calculation of f_+. We found that, although the best-fit values of the Gegenbauer moments $a^\pi_{2,4}$ depend on m_b, the resulting predictions for the form factors and the partial branching fractions are largely independent of the input parameters. This is one of the main results of this letter: the experimental information on the shape of the spectrum reduces the theoretical uncertainty of the prediction for the total branching ratio and hence the extracted value of $|V_{ub}|$. In order to constrain ϕ_π even further, it will be necessary to perform a combined analysis including also other experimental constraints from e.g. the π electromagnetic form factor [12] and the π–γ transition form factor [13].

The determination of $|V_{ub}|$ presented in this letter can be improved in the future from both the experimental and the theoretical side. As for the latter, we would like to stress that the normalisation of f_+ depends on the treatment of f_B, the decay constant of the B meson. We have determined f_B from a QCD sum rule to $O(\alpha_s)$ accuracy, using the same

values for quark mass and quark condensate as in the LCSR for $f_B f_+$, and we have shown, by comparison with the corresponding tree-level sum rules, that the individually large $O(\alpha_s)$ corrections to $f_B f_+$ and f_B cancel in the ratio to a large extent. We have estimated the uncertainty of this procedure to be $\sim 8\%$, which enters the normalisation of the form factor. This overall uncertainty can be reduced by calculating e.g. $O(\beta_0\alpha_s^2)$ corrections to the LCSRs, which is a formidable, but not impossible task. The full $O(\alpha_s^2)$ corrections to f_B are known and actually also reduce the uncertainty coming from m_b [16]. As for the experimental input, smaller bins in q^2 would help to further constrain the shape of the form factor and ultimately avoid the necessity for extrapolation in q^2.

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References

[1] B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0507003.

[2] P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014015 [arXiv:hep-ph/0406232].

[3] M. Wirbel, B. Stech and M. Bauer, Z. Phys. C **29** (1985) 637;
J. G. Körner and G. A. Schuler, Z. Phys. C **38** (1988) 511 [Erratum-ibid. C **41** (1989) 690];
N. Isgur *et al.*, Phys. Rev. D **39** (1989) 799.

[4] P. Ball, JHEP **9809** (1998) 005 [arXiv:hep-ph/9802394];
P. Ball and R. Zwicky, JHEP **0110** (2001) 019 [arXiv:hep-ph/0110115].

[5] V.M. Belyaev, A. Khodjamirian and R. Rückl, Z. Phys. C **60** (1993) 349 [hep-ph/9305348];
A. Khodjamirian *et al.*, Phys. Lett. B **410** (1997) 275 [hep-ph/9706303];
E. Bagan, P. Ball and V.M. Braun, Phys. Lett. B **417** (1998) 154 [hep-ph/9709243];
A. Khodjamirian *et al.*, Phys. Rev. D **62** 114002 (2000) [hep-ph/0001297].

[6] J. Shigemitsu *et al.*, arXiv:hep-lat/0408019;
M. Okamoto *et al.*, Nucl. Phys. Proc. Suppl. **140** (2005) 461 [arXiv:hep-lat/0409116].

[7] D. Becirevic and A. B. Kaidalov, Phys. Lett. B **478**, 417 (2000) [arXiv:hep-ph/9904490].

[8] L. Lellouch, Nucl. Phys. B **479** (1996) 353 [arXiv:hep-ph/9509358];
M. Fukunaga and T. Onogi, Phys. Rev. D **71** (2005) 034506 [arXiv:hep-lat/0408037];
M. C. Arnesen *et al.*, arXiv:hep-ph/0504209.

[9] S. B. Athar *et al.* [CLEO Collaboration], Phys. Rev. D **68** (2003) 072003 [arXiv:hep-ex/0304019];
K. Abe *et al.* [BELLE Collaboration], arXiv:hep-ex/0408145;
B. Aubert [BABAR Collaboration], arXiv:hep-ex/0506064.

[10] P. Ball and A. N. Talbot, JHEP **0506** (2005) 063 [arXiv:hep-ph/0502115].

[11] V. M. Braun and I. E. Filyanov, Z. Phys. C **44**, 157 (1989) [Sov. J. Nucl. Phys. **50**, 511 (1989)].

[12] S. V. Mikhailov and A. V. Radyushkin, Phys. Rev. D **45** (1992) 1754;
A. Schmedding and O. I. Yakovlev, Phys. Rev. D **62** (2000) 116002 [arXiv:hep-ph/9905392];
V. M. Braun, A. Khodjamirian and M. Maul, Phys. Rev. D **61** (2000) 073004 [arXiv:hep-ph/9907495].

[13] A. P. Bakulev, S. V. Mikhailov and N. G. Stefanis, Phys. Lett. B **508** (2001) 279 [Erratum-ibid. B **590** (2004) 309] [arXiv:hep-ph/0103119];
A. P. Bakulev *et al.*, Phys. Rev. D **70** (2004) 033014 [Erratum-ibid. D **70** (2004) 079906] [arXiv:hep-ph/0405062].

[14] S. Hashimoto, arXiv:hep-ph/0411126.

[15] Heavy Flavor Averaging Group (HFAG) (J. Alexander *et al.*),
`http://www.slac.stanford.edu/xorg/hfag/semi/lp05/update.html`

[16] A. A. Penin and M. Steinhauser, Phys. Rev. D **65**, 054006 (2002) [arXiv:hep-ph/0108110];
M. Jamin and B. O. Lange, Phys. Rev. D **65**, 056005 (2002) [arXiv:hep-ph/0108135].